Chapters 22/23: Potential/Capacitance
Tuesday September 20th

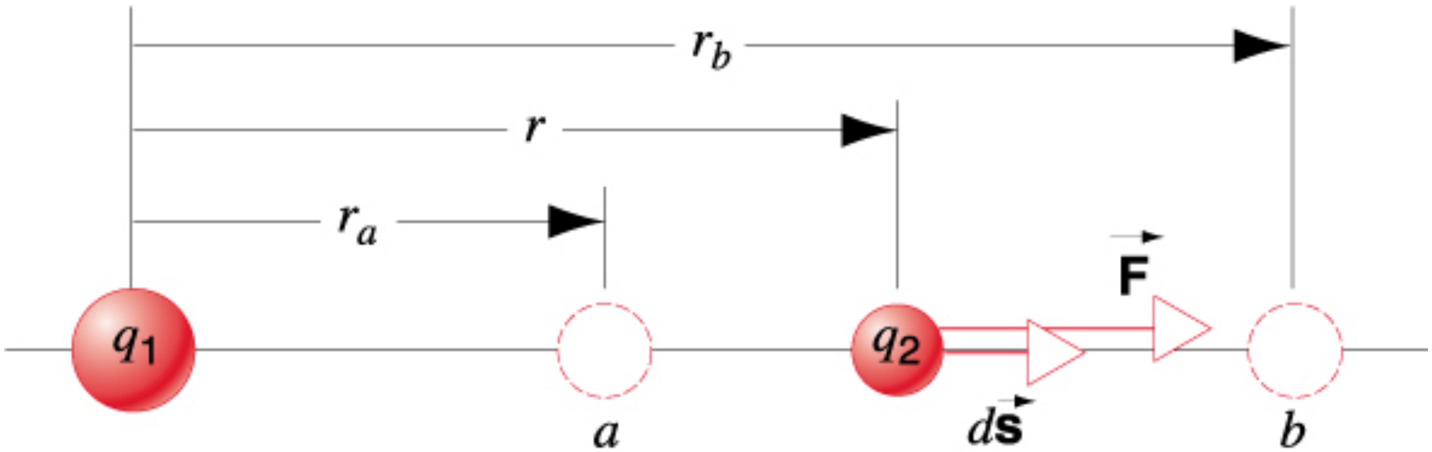
Mini Exam 2 on Thursday:
- Covers Chs. 21 and 22 (Gauss' law and potential)
- Covers LONCAPA #3 to #6 (due this Wed.)
- No formula sheet allowed!!

- Review: Electrostatic potential energy
- Review and continuation: Electrostatic potential
  - Relationship between *V* and *E*
- Capacitance
  - Definition
  - Examples
- Equipotential surfaces
  - Conductors
  - Relationship between *E* and *V*

Reading: up to page 386 in the text book (Chs. 22/23)

Electrostatic Potential Energy

- *The electrostatic (Coulomb) force is conservative.*
- *It is this property that allows us to define a scalar potential energy (one cannot do this for non conservative forces).*



$$\Delta U = -\int_a^b \vec{\mathbf{F}} \cdot d\vec{\boldsymbol{r}} = +\frac{1}{4\pi\varepsilon_o} q_1 q_2 \left(\frac{1}{r_b} - \frac{1}{r_a} \right)$$

- *One can then apply energy conservation (PHY2048).*

Electrostatic Potential Energy

- ***The <u>potential energy</u> is a property of both of the charges, not one or the other.***
- ***If we choose a reference such that $U = 0$ when the charges are infinitely far apart, then we can simplify the expression for the potential energy as follows.***

$$U(r) = -\int_{\infty}^{r} \vec{\mathbf{F}} \cdot d\vec{\boldsymbol{r}} = +\frac{1}{4\pi\varepsilon_o} q_1 q_2 \left(\frac{1}{r} - \frac{1}{\infty} \right) = \frac{1}{4\pi\varepsilon_o} \frac{q_1 q_2}{r}$$

Again, the sign of U is not a problem. It is taken care of by the signs of the charges q_1 and q_2.

The Electrostatic Potential

- *We define a new quantity known as the Electrostatic Potential* ***V***, *simply by dividing out the test charge* q_o:

i.e., $$U = q_o V$$

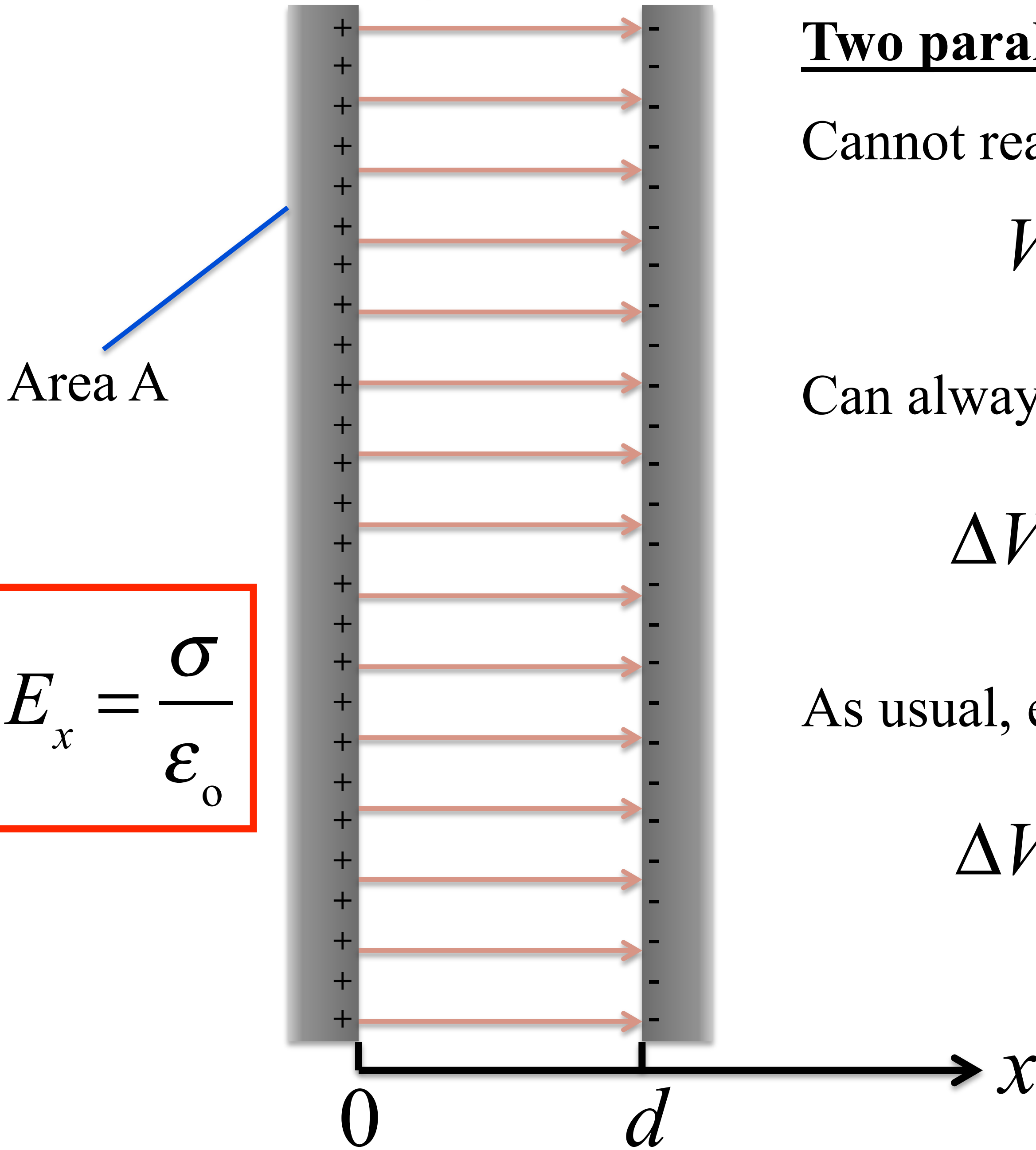
$$\Rightarrow \Delta V = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{r}} = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

- ***This 'scalar potential' depends only on the details of the source charge distribution*** **(in this case, q).**

$$V(r) = -\int_\infty^r \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{r}} = \frac{q}{4\pi\varepsilon_o}\frac{1}{r} = k\frac{q}{r}$$

- ***The absolute value of the potential is not important. As we shall see, it is only potential differences that really matter.***

Calculating Potential Difference from E



$$E_x = \frac{\sigma}{\varepsilon_o}$$

Two parallel conducting plates

Cannot really use:

$$V(r) = k\frac{q}{r}$$

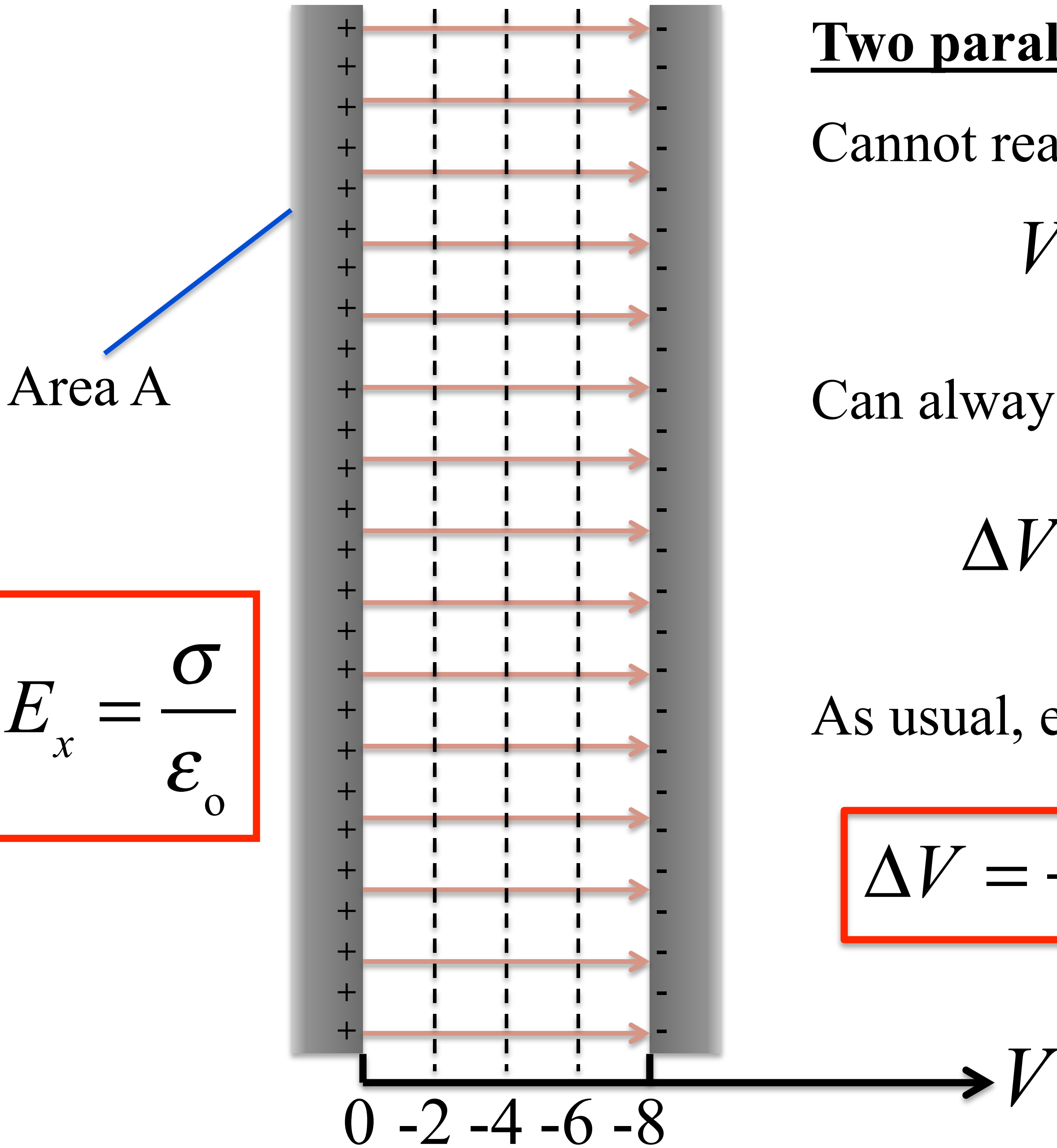
Can always use definition:

$$\Delta V = -\int_{r_1}^{r_2} \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{r}}$$

As usual, exploit the symmetry:

$$\Delta V = -\int_0^x E_x \, dx$$

Calculating Potential Difference from *E*



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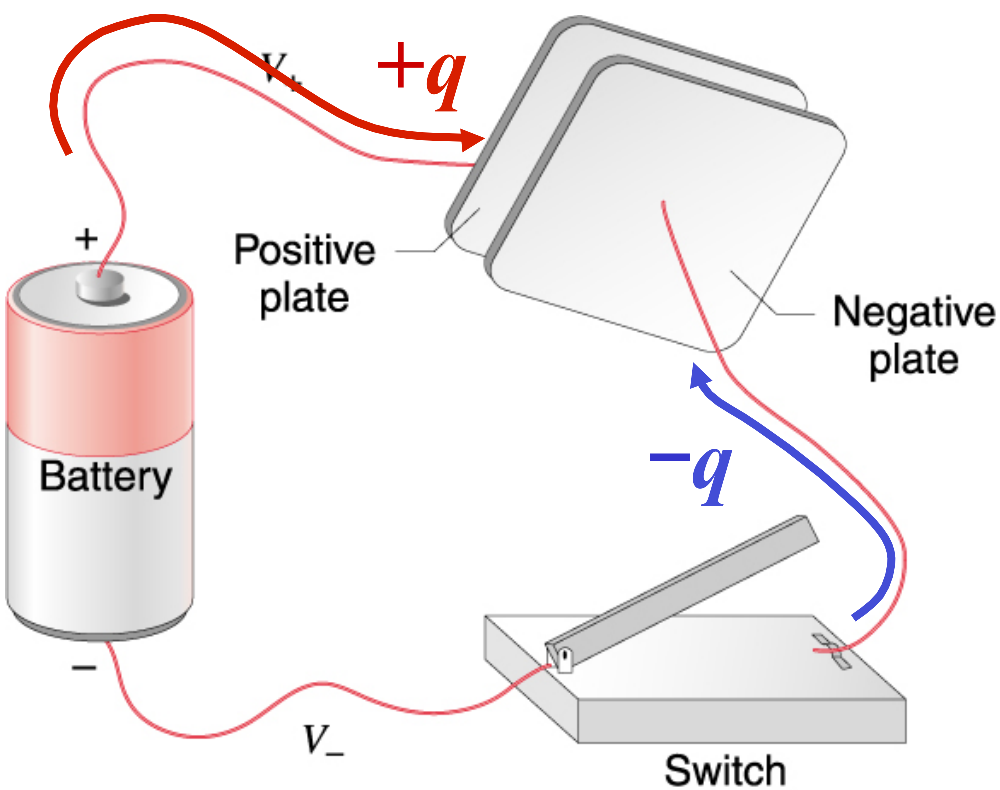
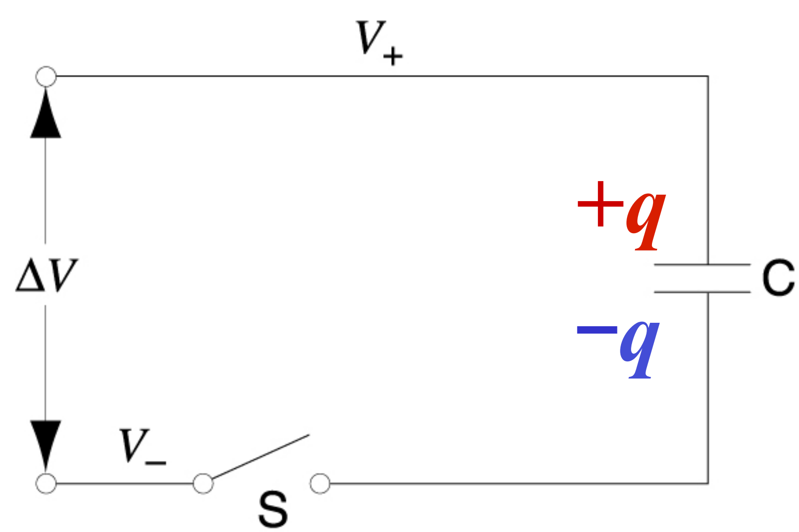
As usual, exploit the symmetry:

$$\Delta V = -E_x \Delta x = -\frac{\sigma}{\varepsilon_o} d$$

Capacitors

- Used to store energy in electromagnetic fields [in contrast to batteries (chemical cells) that store chemical energy].
- Capacitors can release electromagnetic energy much, much faster than chemical cells. They are thus very useful for applications requiring very rapid responses.





Capacitors

- The transfer of charge from one terminal of the capacitor to the other creates the electric field.
- Where there is a field, there must be a potential difference, i.e., a voltage difference between terminals.
- This leads to the definition of capacitance *C*:

$$Q = C\Delta V$$

- *Q* represents the magnitude of the excess charge on either plate. Another way of thinking of it is the charge that was transferred between the plates.

SI unit of capacitance: 1 farad (F) = 1 coulomb/volt

(after Michael Faraday)

Capacitances more often have units of picofarad (pF) and microfarad (μF)

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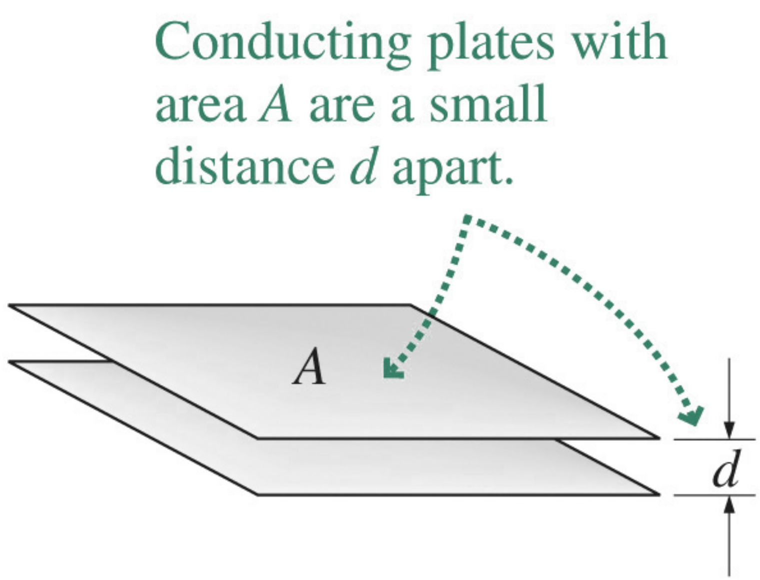
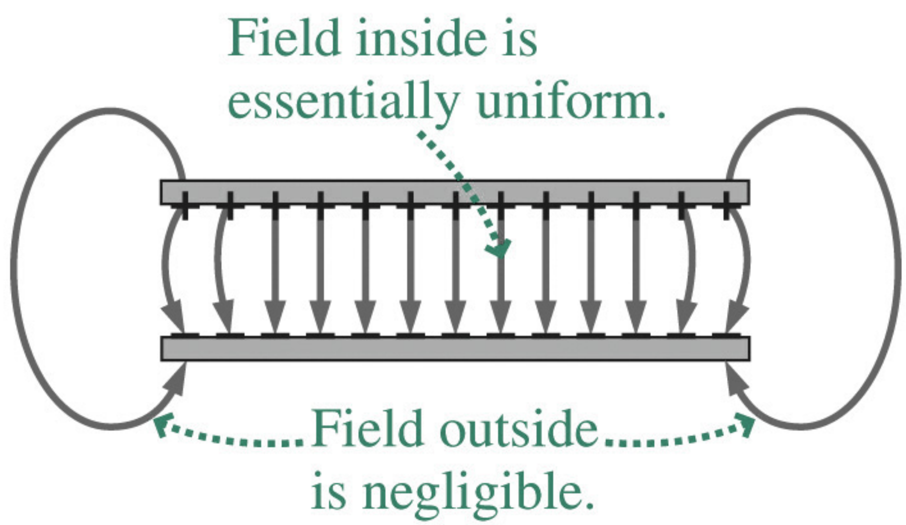
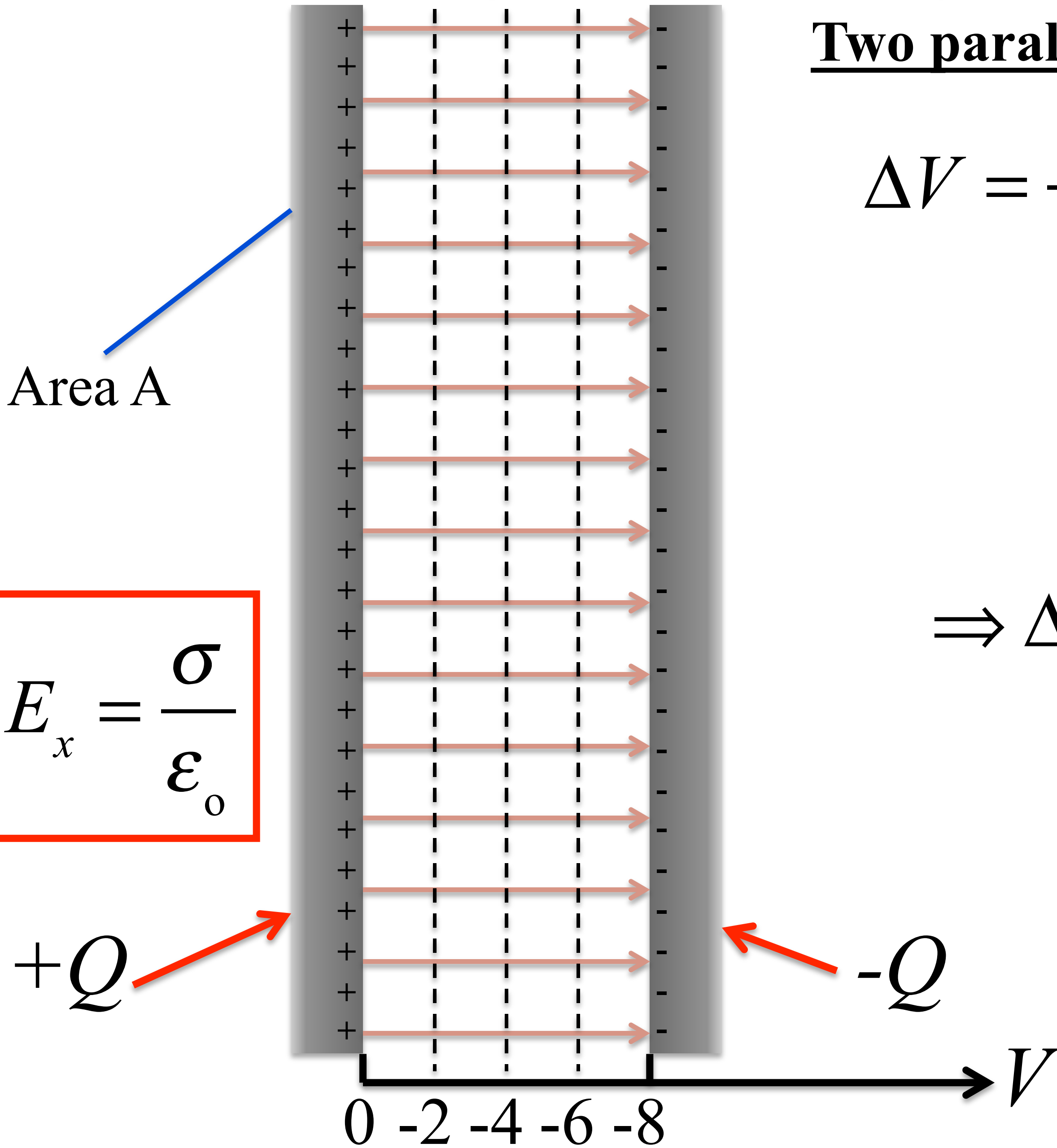




Our Example Involving Parallel Plates



$$E_x = \frac{\sigma}{\varepsilon_o}$$

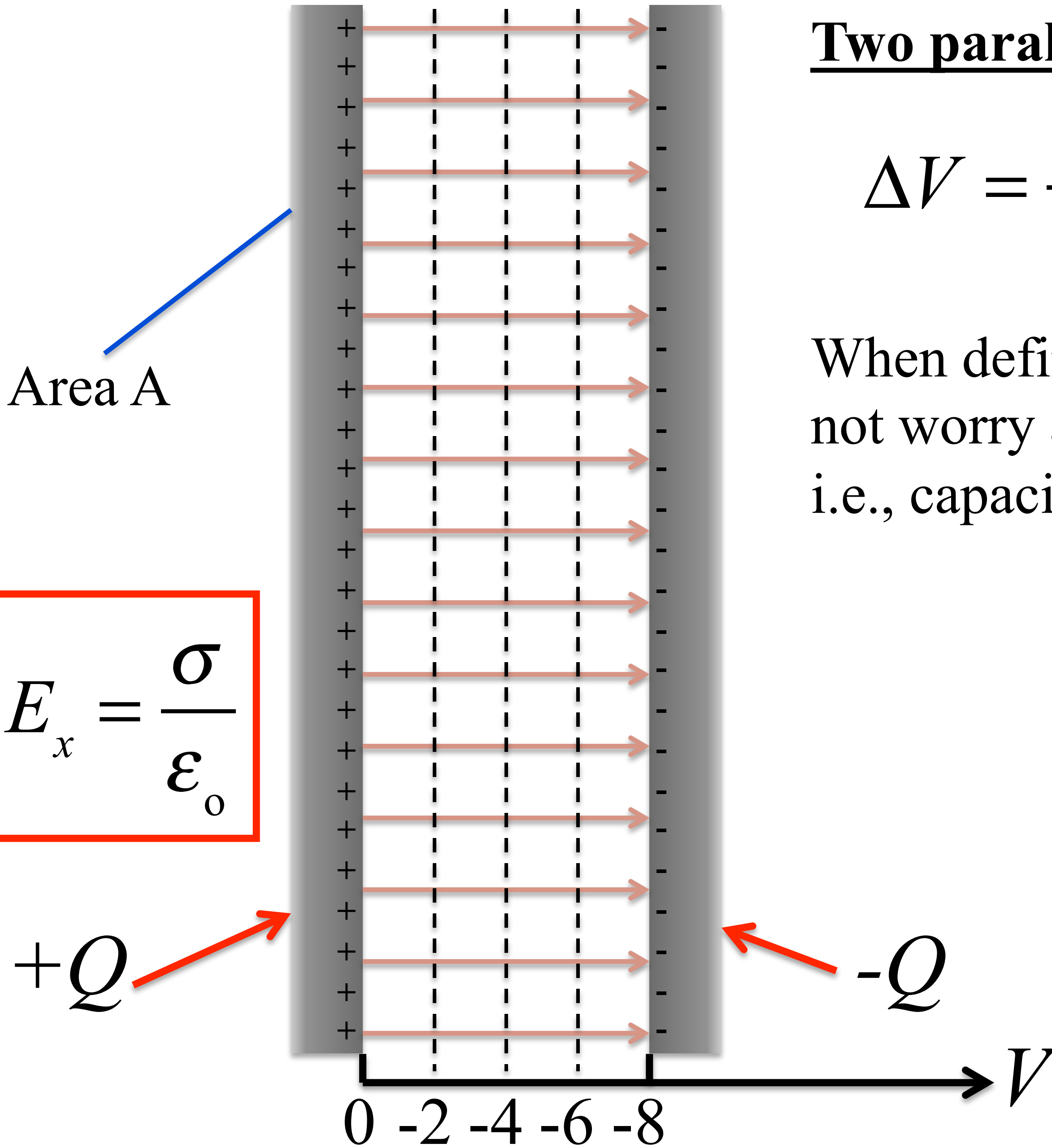
Two parallel conducting plates

$$\Delta V = -E_x \Delta x = -\frac{\sigma}{\varepsilon_o} d$$

$$\sigma = \frac{Q}{A}$$

$$\Rightarrow \Delta V = -\frac{Q}{A\varepsilon_o} d$$

Our Example Involving Parallel Plates



$$E_x = \frac{\sigma}{\varepsilon_o}$$

Two parallel conducting plates

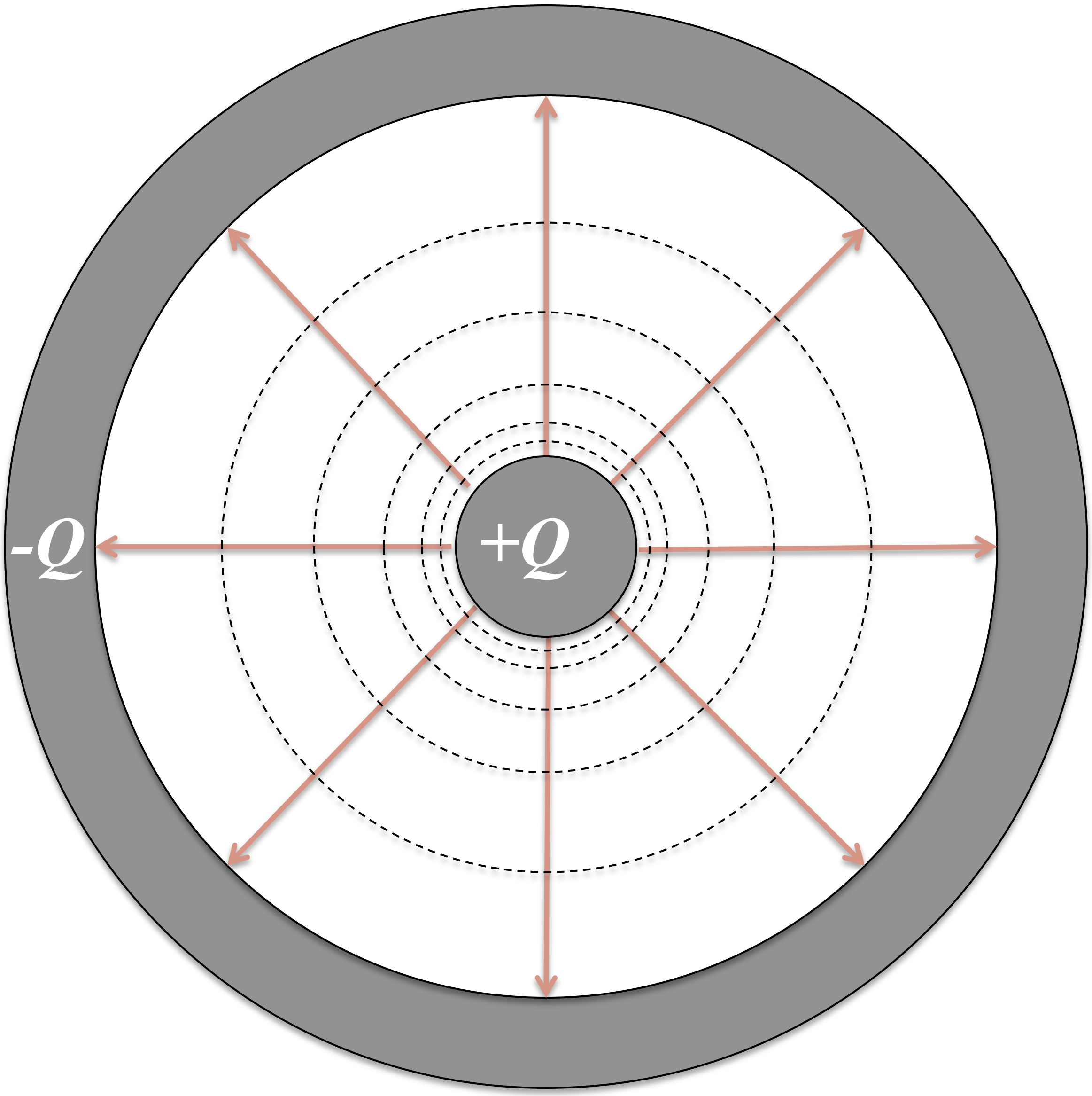
$$\Delta V = -E_x \Delta x = -\frac{\sigma}{\varepsilon_o} d$$

When defining capacitance, we do not worry about sign of potential, i.e., capacitance is always positive

$$\Delta V = Q\frac{d}{A\varepsilon_o} = \frac{Q}{C}$$

$$\Rightarrow C = \frac{\varepsilon_o A}{d}$$

Another Example: Concentric Spheres



Can use both

$$V(r) = k\frac{q}{r}$$

and

$$\Delta V = -\int_{r_1}^{r_2} \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{r}}$$

$$\vec{\boldsymbol{E}}(r) = k\frac{q}{r^2}\hat{\boldsymbol{r}}$$

Some sophisticated vector calculus

The fundamental theorem of calculus:

$$\int_{x_1}^{x_2} \left(\frac{df}{dx} \right) dx = f\left(x_2\right) - f\left(x_1\right)$$

Recall the 1D example involving the parallel plates:

$$\Delta V = -\int_{r_1}^{r_2} \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{r}} = \int_{x_1}^{x_2} \left(-E_x\right) dx = V_2 - V_1$$

$$\Rightarrow E_x = -\frac{dV}{dx} \approx -\frac{\Delta V}{\Delta x}$$

Potential Does Not Kill You

Some more sophisticated vector calculus

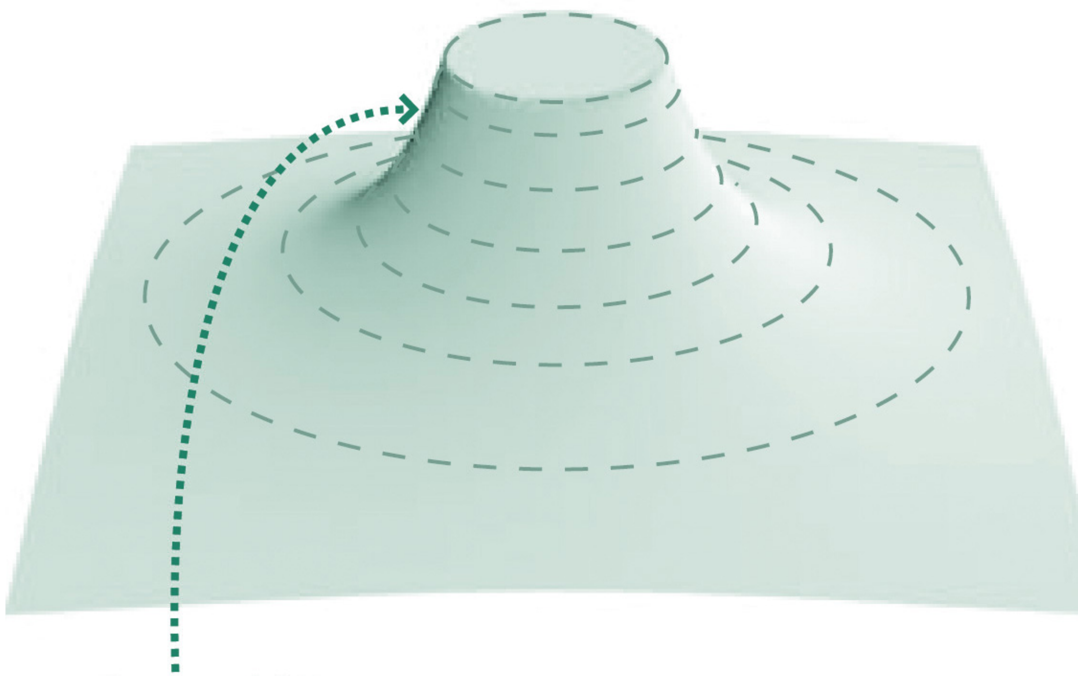
The fundamental theorem of calculus:

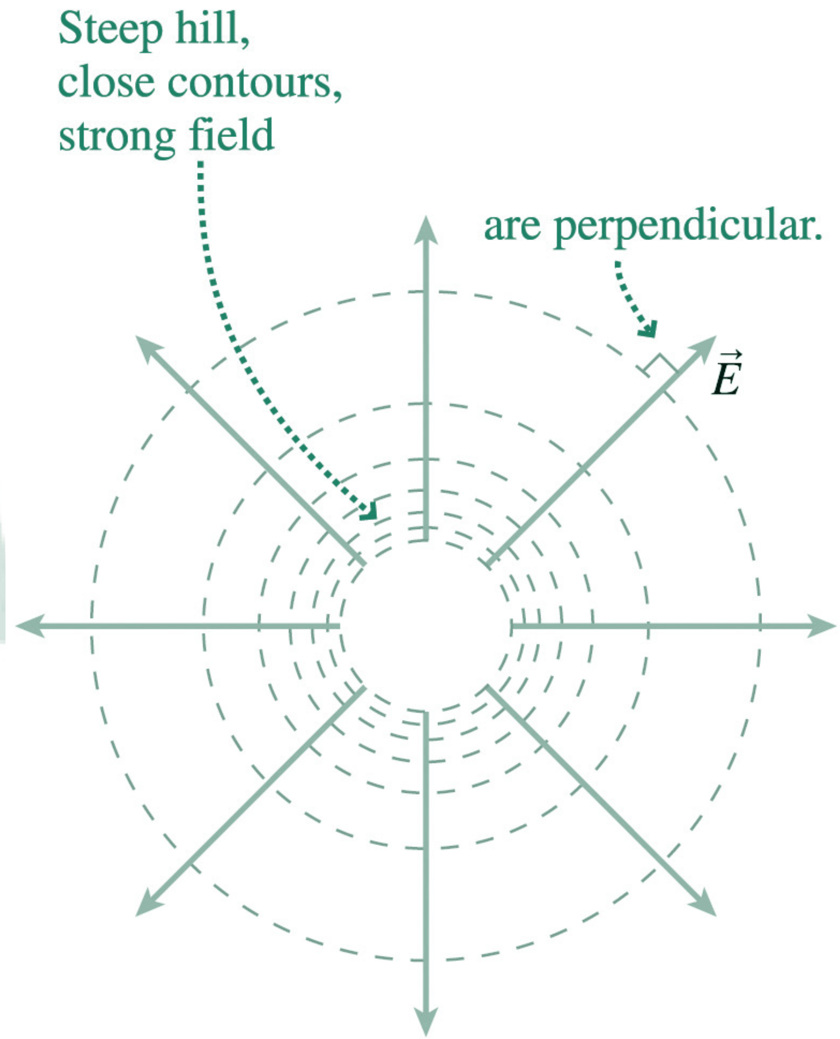
In 3D:

$$\Delta V = \int_{\vec{\mathbf{r}}_1}^{\vec{\mathbf{r}}_2} \nabla V \cdot d\vec{\mathbf{r}} = V\left(\vec{\mathbf{r}}_2\right) - V\left(\vec{\mathbf{r}}_1\right)$$

$$\boxed{\vec{\boldsymbol{E}} = -\nabla V = -\left(\frac{\partial V}{\partial x}\hat{\boldsymbol{i}} + \frac{\partial V}{\partial y}\hat{\boldsymbol{j}} + \frac{\partial V}{\partial z}\hat{\boldsymbol{k}}\right)}$$

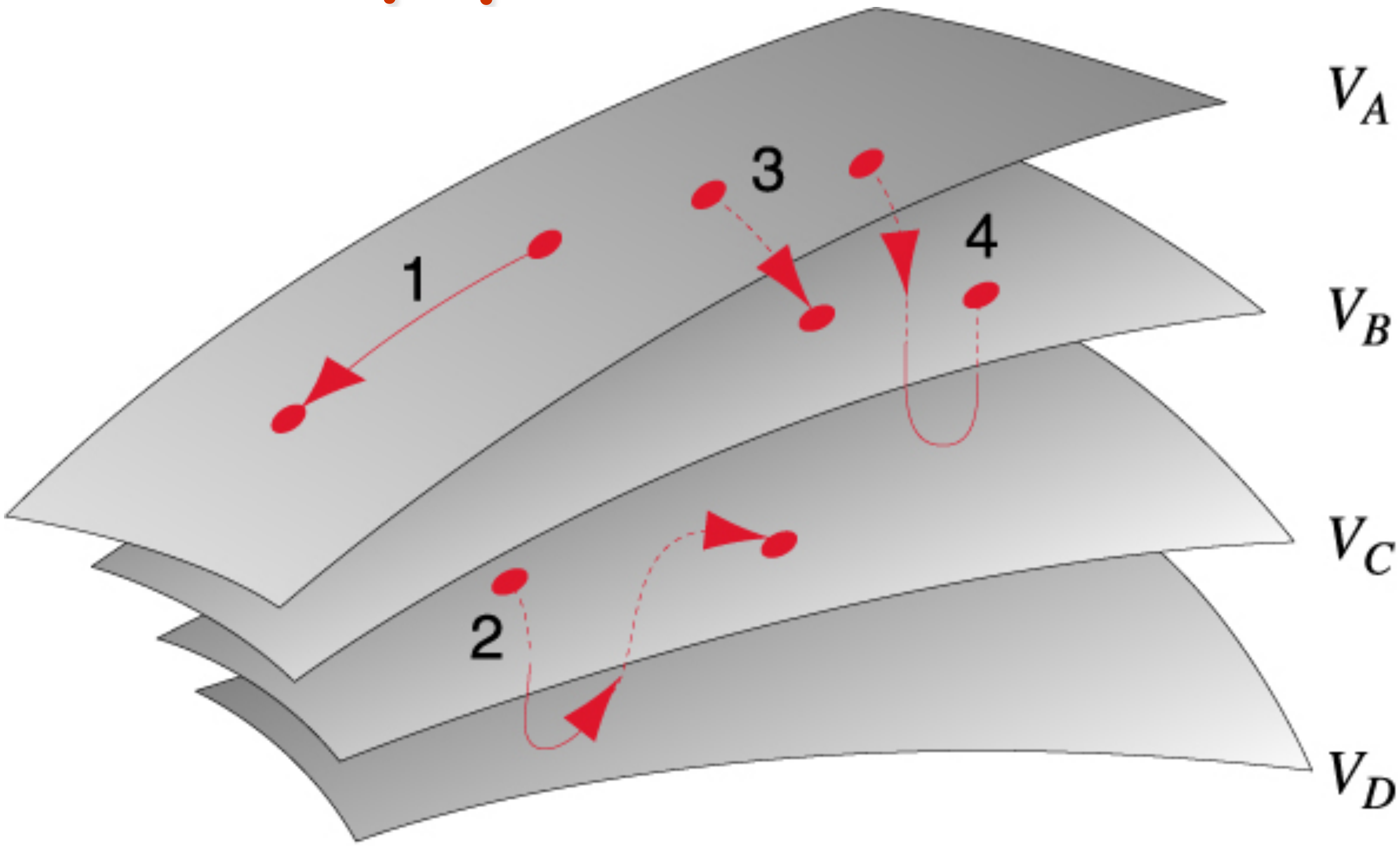
Equipotential lines/surfaces





$$E_r \approx -\frac{\Delta V}{\Delta r}$$

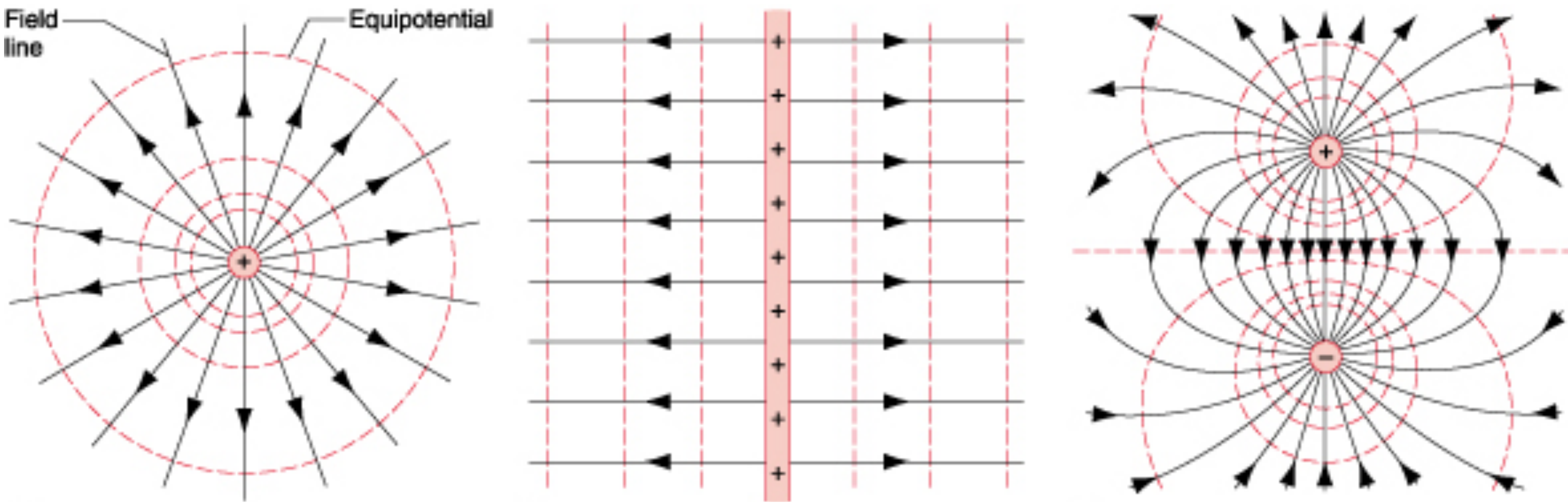
Equipotential surfaces



$$W_{field} = -q\Delta V = -q\int_a^b \vec{\mathbf{E}} \cdot d\vec{r} = -q\left(V_b - V_a\right)$$

- ***E-field lines perpendicular to equipotential surfaces***
- ***Field does positive work when q accelerated by field***

Equipotential surfaces



Here are some example surfaces including field lines (point charge, infinite charged plane and a dipole).

- ***By spacing the equipotential surfaces by the same potential difference (ΔV), one can get a feel for the electric field strength ($E = -dV/dr$), i.e. the closer the spacing, the stronger the field.***